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Investigationand Comparison of Light Propagation in Two Graded Photonic Crystal Structures

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Abstract: In this paper, we study two different Graded Index (GRIN) photonic crystal (PC) structures which are named as structure type I and type II. The PC structures are made of the square rod in an air background. To design a GRIN PC structure the lattice constant has been altered in the direction transverse to propagation. We investigated focusing effect and waveguiding behavior of electromagnetic waves propagating throughout the GRIN structures. Wide incident beam with a Gaussian profile is illuminated to the both GRIN structures. In the case of structure type I, the incident wave bend toward the central part of the structure and after getting out of it focuses on the narrow area. The designed structure shows the strong focusing effect and increase normalized intensity and decreases the width of the outgoing wave. In the case of structure type II, the incoming beam after traveling short distance becomes distracted to the central part and remains confine in the middle of the structure. The exiting wave can be coupled to a photonic crystal waveguide. Plane wave expansion method has been used for analyzing dispersion relation. Propagation of electromagnetic wave within the graded structure has been simulated using finite-difference time-domain method.

Keywords: photonic crystals, graded index, effective refractive index, plane wave expansion.

1. Introduction

Photonic crystals (PCs) are periodic structures in which dielectric constant periodically vary in one, two or three dimensions. Great attention to the PCs start by the pioneering work has been done by John and Yablonovitch [1-2]. PCs are the structure that effectively controls light propagation direction. One of

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peculiar characteristic of the PCs is that they exhibit photonic band gaps (PBGs). Therefore the PCs are able to block the electromagnetic wave from propagating through the structure while the frequency falls in the PBG. Completely periodic PC can be broken by introducing point or line defect. As a result, artificially created modes can be concentrated in a small area or guide through the waveguides [3-4]. The ability to control light propagation direction can be improved by introducing the idea of graded index (GRIN) medium. The term graded index are used for describing an inhomogeneous media in which refractive index can vary point by point. A GRIN PC structure can be created by gradually varying PCs parameters. There are several ways for producing an index gradient such as changing lattice constant, radii of the rods or dielectric constant. Here we have altered the lattice spacing in order to produce a GRIN PC structure with refractive index gradient. Two different GRIN PC structure have been designed. The electromagnetic wave behavior through these structures is investigated. The PC structure under study is made of the square rod in the dielectric background. To the best of our knowledge, this is the first time that the noncircular element such as square shape element has been used in designing GRIN PC structure. First the focusing effect has been shown in the structure type I and as a result, the structure can work very well as a lens. Second, the waveguiding behavior has been shown through the structure type II so it can be used as a coupler in order to increase coupling efficiency.

2. Theory

For determining photonic band structure of periodic dielectric structure we solve the Maxwell equations by the plane wave expansion (PWE) method which has been studied in several papers [5-7]. Maxwell equation for magnetic field can be written as following [8]

$$\vec{\nabla}\times\left[\frac{1}{\varepsilon(\vec{r})}\vec{\nabla}\times\vec{H}(\vec{r})\right]=\frac{\omega^2}{c^2}\vec{H}(\vec{r}). \quad (1)$$

Where c is light velocity in a vacuum, ω is the angular frequency of light and $\varepsilon(\vec{r})=\varepsilon(\vec{r}+\vec{R})$ is a position dependent dielectric function which is periodic with respect to the lattice vector $\vec{R}$. The photonic band structure can be obtained by solving equation (1) by PWE method [9]. From Bloch theorem, the inverse of dielectric function can be expanded by the group of plane waves [10]

$$\frac{1}{\varepsilon(\vec{r})}=\sum_{\vec{G}}\chi(\vec{G}).\exp(j\vec{G}.\vec{r}) \quad (2)$$

Where $\chi(\vec{G})$ is Fourier transform of inverse $\varepsilon(\vec{r})$ which is a key parameter in determining photonic band structure and is a function of primitive lattice vector and will be determined as following

$$\chi(\vec{G}) = \frac{1}{V_0}\int_{V_0}\frac{1}{\varepsilon(\vec{r})}\exp(j\vec{G}.\vec{r})d\vec{r} \tag{3}$$

Where V_0 describes volume of PC unit cell. Theoretical investigation of 2D PC is much easier because the main equation can be solved for two polarizations separately. These polarizations are TM and TE. In the first case, an electric field is perpendicular to the PC plane and the second case will occur the same for the magnetic field. Because of wave propagation in periodic structures we can expand the wave functions in terms of wave vector [10]

$$\vec{H}(\vec{r}) = \sum_{\vec{G}}\sum_{\lambda=1}^{2} h_{\vec{G},\lambda}\hat{e}_\lambda e^{i(\vec{k}+\vec{G}).\vec{r}} \tag{4}$$

Where $\vec{k}$ is the wave vector in the first Brillouin zone and $\vec{G}$ is a 2D reciprocal lattice vector, $\hat{e}_\lambda(\lambda = 1,2)$ are orthogonal unit vectors perpendicular to $\vec{k}+\vec{G}$. By replacing equations (2) and (4) in (1) we reach the following equations

$$\sum_{\vec{G}} H_{\vec{G},\vec{G}'}\begin{pmatrix} h_{\vec{G}',1}\\ h_{\vec{G}',2}\end{pmatrix} = \frac{\omega^2}{c^2}\begin{pmatrix} h_{\vec{G},1}\\ h_{\vec{G},2}\end{pmatrix} \tag{5}$$

Where

$$\vec{H}_{\vec{G},\vec{G}'} = \left|\vec{k}+\vec{G}\right|\left|\vec{k}+\vec{G}'\right|\chi(\vec{G}-\vec{G}')\begin{bmatrix}\hat{e}_2.\hat{e}'_2 & -\hat{e}_2.\hat{e}'_1\\ -\hat{e}_1.\hat{e}'_2 & +\hat{e}_1.\hat{e}'_1\end{bmatrix} \tag{6}$$

In 2D PC for all $\vec{G}$, $\vec{k}+\vec{G}$ is placed in a x-y plane, therefore $\hat{e}_2.\hat{e}'_1 = \hat{e}_1.\hat{e}'_2 = 0$. For incoming light perpendicular to the rod axis, equation (6) will separate to the two scalar problems corresponding to the polarizations TE and TM. In the case TM polarization, $h_{\vec{G},1} = 0$ and for all $\vec{G}$, the eigenvalue problem will be as follow

$$\sum_{\vec{G}'}\left|\vec{k}+\vec{G}\right|\left|\vec{k}+\vec{G}'\right|\chi(\vec{G}-\vec{G}')h_{\vec{G}',2} = \frac{\omega^2}{c^2}h_{\vec{G},2}. \tag{7}$$

This equation will be known as “Master equation” for 2D PC. The main peculiarity of the equation (7) is the absence of the coordinate dependence. All the variables depend on the reciprocal lattice vector. We have an infinite number of linear equations that can be truncated to some quite low number of equations depending on the accuracy required. This is the moment of plane wave expansion method [10].

3. Geometrical Structure

In this paper, we consider two different GRIN PC structures. Our aim is to show the focusing effect and waveguiding behavior by the designed structures. The unmodified PC structure is composed of the square rod in air background in a square lattice. In order to design the structure with a graded refractive index, we have altered lattice spacing in the direction transverse to propagation. The lattice constant is minimum in the central part of the structure and increases toward the edges which reach the maximum value. The schematic representation of two structures has been shown in Figure 1.

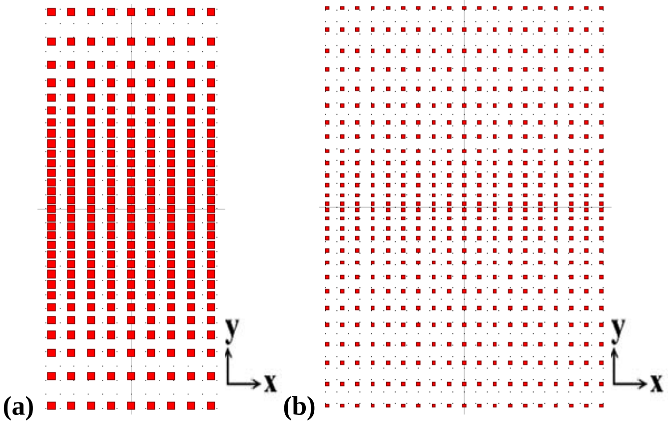


Fig. 1. The schematic representation of (a) GRIN PC structure type I (b) GRIN PC structure type II, both structures are composed of square rods in an air background

The structure type I is made of the square rod with the length equals to $0.37a$ which a is fixed lattice constant in the x-direction. The structure is composed of 9 columns in the x-direction and 31 rows in the y-direction. Lattice constant in the y-direction is $0.44a$ and $1.5a$ respectively in the center and at the edges of the structure. The refractive index of rods is 3.13 and also is in the structure type II. The structure type II is made of the square rod in which the length equals $0.22a$ with 20 columns and 28 rows in x and y-direction respectively. Lattice constant in the y-direction in the central part is minimum and is $0.44a$ and at the edges is maximum and equal $2.0a$.

The plane wave expansion method has been carried out for calculating photonic band structure. Dispersion diagram along the ΓX direction for the first band has been plotted for several values of lattice constant in figure 2(a).The effective refractive index of the PCs can be calculated from the relation

$$N_g = c / \nabla_k \omega(k). \tag{8}$$

Where ω is angular frequency and $\vec{k}$ is wave vector. Indeed, the effective refractive index will be calculated from the slope information of the dispersion diagram. Figure 2(b) shows the effective refractive index diagram for both structures. For lower frequencies, curves are closely spaced and provide a slight variation in effective refractive indices. Each curve enters the cut-off region at different frequencies that are longer comparative to the rod in air designed structures.

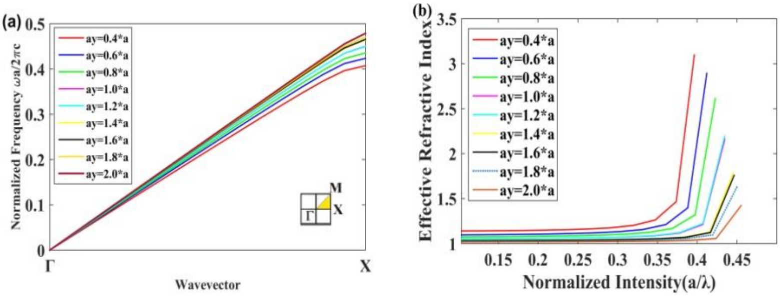
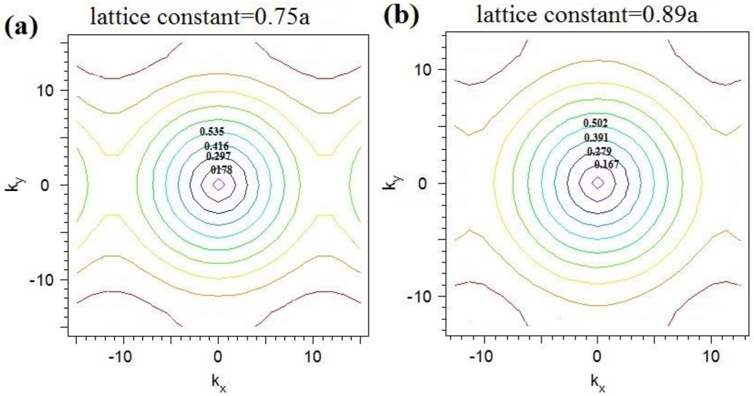


Fig. 2. (a) Dispersion diagram in ΓX direction of the first band for several values of lattice constant (b) the effective refractive index in ΓX direction

The beam bending can be achieved by modification of the parameter of the PC structure. Indeed, the GRIN PC structures are able to curve the light propagation direction. The GRIN PC structures are not strictly periodic but when the refractive index gradient is small we can calculate optical characteristic from basic PC structure. The beam propagation through the GRIN PC can be explained by the study of iso-frequency curve (IFC) in wave vector space.



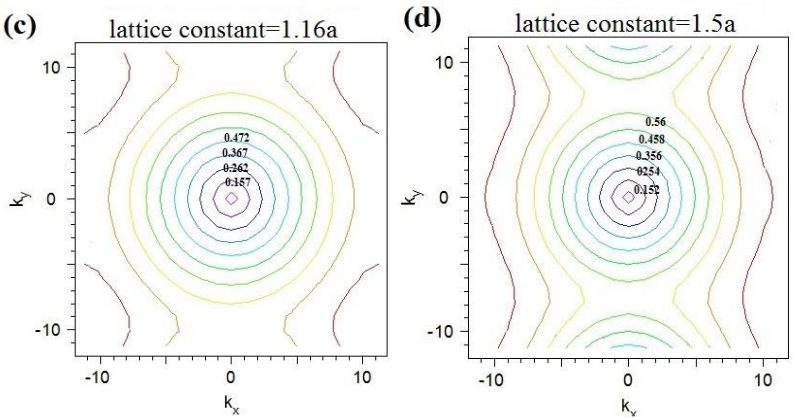


Fig. 3. Iso-frequency curves for different value of lattice constant (a) 0.75a (b) 0.89a (c) 1.16a (d) 1.5a

Figure 3 shows the iso-frequency curves that have been plotted for the first band of perfect PC which is made of square rod in air background with different values of lattice constants. The direction of light propagation can be explained by IFC plot which is calculated by the plane wave expansion method. The curving the flow of light is a result of gradual modification of GRIN PC structural parameter in which group velocity will be location dependent. For studying focusing behavior through the GRIN structures the frequency of incident beam has been selected 0.18 with TM polarization. As can be seen from the Figure 3, in the working frequency the IFCs are circle. When these curves are circular, the GRIN structure can be considered as a homogeneous medium.

4. RESULTS OF SIMULATION

As a first step, we want to show the focusing effect of the GRIN PC lens type I. A source with a Gaussian profile of width 12a is located in front of the structure. Normalized frequency, ωa/2πc, equals to 0.18 has been chosen with the TM polarization in which electric field is parallel to the rod axis. A Gaussian beam is excited the lens and after traveling a short distance, the beam is bent toward the center of the structure and its width is reduced. As Finite Different Time Domain (FDTD) simulation in Figure 4(a) shows, after getting out of the structure the beam focuses to a small area and a real focal point is created. Figure4 (b) represents spatial electric field distribution throughout the lens type I.

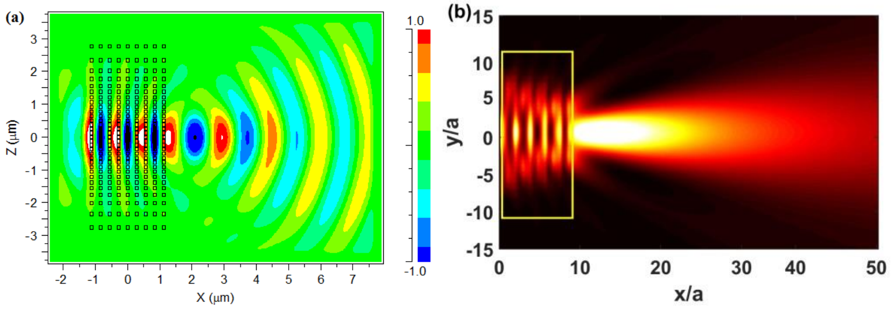


Fig. 4. (a) A representation of focal point creation at the output of the GRIN PC lens type I (b) The field propagation throughout the GRIN PC lens type I

Next, the normalized frequency in the focal point position has been plotted in Figure 5. As can be seen from the figure, the width of the input beam is reduced and the normalized intensity at the focal point is increased after crossing the lens. For precise analysis, the full-width at half maximum (FWHM) has been calculated. The FWHM value changes from 2.26λ for input beam to 0.9λ for output beam. The results show that a Gaussian beam of width 12a was distracted toward the central part of the structure then after crossing the lens focuses to a focal point. Normalized intensity plot shows that the GRIN PC type I reduce the width and increase the intensity of the incoming wave.

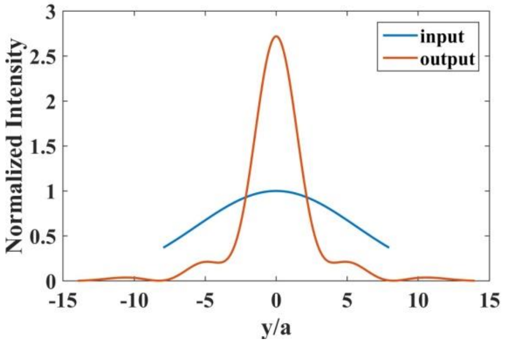


Fig. 5.Normalized Intensity profile at focal point position at the output of the GRIN PC lens type I

As a second step, we want to show waveguiding behavior in the designed structure type II. A continued wave with a Gaussian profile of width 18a is illuminated to the structure type II from the left-hand side. The normalized frequency, ωa /2πc, of the input beam is fixed at 0.44 with TM polarization. Figure (6) shows FDTD simulation of the electric field propagation in the structure.

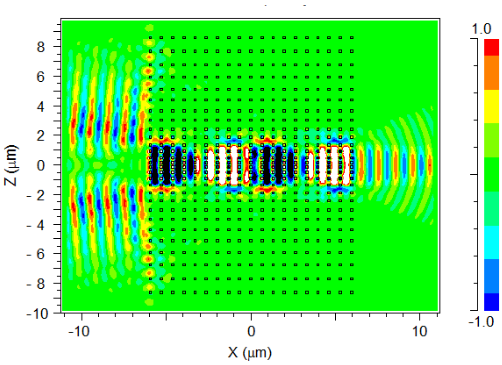


Figure 6.*The field propagation through the GRIN PC lens type II*

As can be seen from the figure, wide incident beam bends toward the central part of the high refractive index and its width reduces and remains confined in the middle of the structure. The output beam can be coupled to a waveguide and worked as a coupler with a high efficiency because a waveguide can be adopted to output beam in a manner that the central output beam sends to the waveguide and propagate through it.

5. Conclusion

In this paper, we have shown focusinge ffect and waveguiding behavior throughout two different GRIN PC structures, made of square rods in air. The gradient of refractive index was achieved by gradually varying the lattice constant in the direction transverse to propagation. The results show that lens type I exhibit strong focusing effect. Indeed, curving the light path throughout the lens relies on gradual modifications of GRIN PC structural parameter. Also, we constructed a GRIN PC structure which can confine the light to a narrow area and actually can work as an intermediate element such as a coupler. We have demonstrated that structure type II can confine wide incident beam to the central part of the structure that can be coupled to a PC waveguide.

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